FAR BEYOND

MAT122

Composition of Functions

Stony Brook University

Composite Function - Intro

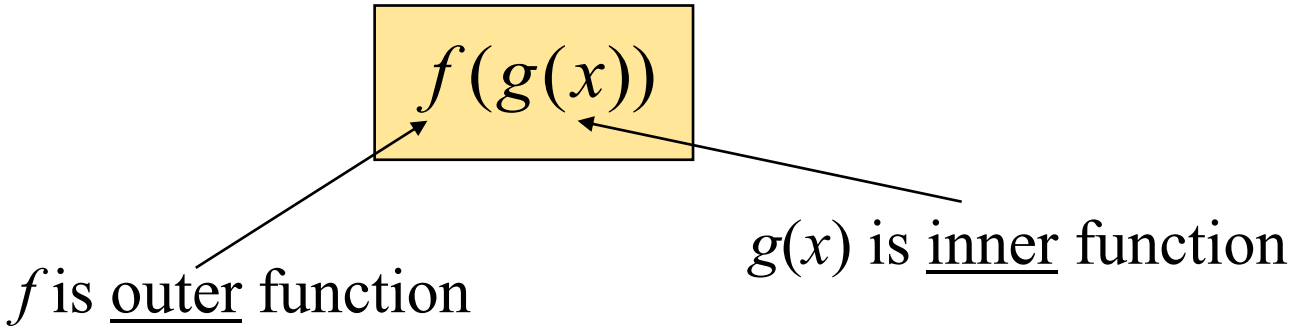
Composite function := a function nested within another function

Ways to denote:

given functions $f(x),\ g(x)$

$(f \circ g)(x)$ or $f \circ g$ or preferred $f(g(x))$

"f of g" or "f composed with g" or "g nested into f"



Composing Functions

ex. given: $f(x) = x^2 - 2x + 6 \quad g(x) = 3x - 4$

1st, rewrite in preferred notation

substitute in g

$$(f \circ g)(x) = f(g(x)) = f(3x-4)$$

plug into f $= (3x-4)^2 \quad -2(3x-4) + 6$

Do: FOIL Do: distribute

$$= 9x^2 - 24x + 16 - 6x + 8 \quad +6$$

combine like terms $= \boxed{9x^2 - 30x + 30}$

see the result when the order is switched…

$$(g \circ f)(x) = g(f(x)) = g(x^2 - 2x + 6)$$

plug into g $= 3(x^2 - 2x + 6) - 4$

distribute $= 3x^2 - 6x + 18 - 4$

combine like terms $= \boxed{3x^2 - 6x + 14}$

it's possible to nest a function into itself…

$$(g \circ g)(x) = g(g(x)) = g(3x-4)$$
$$= 3(3x-4) - 4$$
$$= 9x - 12 - 4$$
$$= \boxed{9x - 16}$$

Try $(f \circ f)(x)$ on your own for extra practice.

$\boxed{\text{(Answer: } x^4 - 4x^3 + 14x^2 - 20x + 30)}$

Evaluating Composed Functions

ex. given: $f(x) = x^2 - 2x + 6,\ g(x) = 3x - 4$ same as previous slide

find $(f \circ g)(1)$

Method #1

know from previous slide that

$(f \circ g)(x) = 9x^2 - 30x + 30$

then…

$(f \circ g)(1) = 9(1)^2 - 30(1) + 30$

$= 9 - 30 + 30$

$= \boxed{9}$

Method #2

evaluate inner function first

$= f(g(1))$

$g(1) = 3(1) - 4$

$= -1$

$= f(-1)$

$= (-1)^2 - 2(-1) + 6$

$= 1 + 2 + 6$

$= \boxed{9}$

Composing Functions Containing Fractions

ex. given: $f(x)=\dfrac{2}{x-1}$ $g(x)=\dfrac{3}{x}$ Find $f\circ g$ and $g\circ f$. Simplify.

$$(f\circ g)(x)=f(g)=f\left(\frac{3}{x}\right)$$

multiply by LCD $=\dfrac{2}{\frac{3}{x}-1}\cdot\dfrac{x}{x}$

distribute x to both terms

$$=\frac{2x}{\frac{3x}{x}-1x}$$

$$=\boxed{\frac{2x}{3-x}}$$

$$(g\circ f)(x)=g(f)=g\left(\frac{2}{x-1}\right)$$

$$=\frac{3}{\frac{2}{x-1}}\cdot\frac{x-1}{x-1}$$

$$=\boxed{\frac{3(x-1)}{2}}$$

or $\dfrac{3x-3}{2}$

Decomposing Functions

Opposite of composition. Identifying inner and outer functions.

ex. split $h(x)$ into two separate functions assuming $h(x) = f(g(x))$

where $h(x) = (3x^2 - 4x + 1)^3$

$\longleftarrow$ inner f $\longrightarrow$

$g(x) = 3x^2 - 4x + 1$

what's left? $f(x) = x^3$ outer function

Do: decompose $h(x) = |2x - 5|$

ex. decompose $h(x) = \sqrt[3]{x^2 + 1}$

$g(x) = x^2 + 1$

$f(x) = \sqrt[3]{x}$

Do: decompose $h(x) = \dfrac{1}{4x + 7}$